Quantum Effect on Collisions in Electron-positron Plasmas

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Quantum effects on the electron-positron scattering are investigated in electron-positron plasmas. The corrected Kelbg potential, taking into account the quantum effects, is applied to describe the electron-positron interaction potential in electron-positron plasmas. The Born approximation is considered to obtain high-energy electron-positron scattering cross sections. The results show that the differential electron-positron scattering cross sections increase with increasing thermal de Broglie wavelength, i. e., decreasing plasma temperature. The differential electron-positron scattering cross sections decreases with increasing collision energy. It is also found that the quantum effects on the differential scattering cross section are small for forward and backward scattering angles.

Key words: Electron-positron Plasmas; Electron-positron Collisions.

Atomic collision processes [1 - 4] in plasmas have been of great interests since these processes can be used for plasma diagnostics. Collision processes involving positrons and electrons have received much attention since these processes have many applications in atomic, plasma physics, and astrophysics [5, 6]. Electron-positron collisions can also contribute to the collective effects for the bremsstrahlung in the case of electron-positron collisions. Such mass-symmetrical plasma systems may be observed in laboratory [7, 8] and astrophysical [5, 6] plasmas. In weakly coupled plasmas, the collision processes have been investigated using the Debye-Hückel potential with various methods depending on the collision systems [9, 10]. The plasmas described by the Debye-Hückel model are called ideal plasmas since the average energy of interaction between particles is small compared to the average kinetic energy of the particles [11]. However, in the region of partial degeneration and strong coupling, the interaction potential differs from a pure Coulomb or screened Coulomb potential because of the strong coupling and quantum effects of nonideal particle interaction. In the present paper we investigate quantum effects on high-energy electron-positron collisions in a mass-symmetrical pair plasma. An effective potential model called the corrected Kelbg potential [12], including the classical effect as well as the quantum-mechanical effect such as the Heisenberg principle and the Pauli exclusion principle is applied to describe the interaction potential in electron-positron plasmas. The first-order Born approximation is applied to obtain the differential scattering cross section as a function of the thermal de Broglie wavelength, scattering angle, and collision energy.

For a potential scattering, the differential scattering cross section $\mathrm{d}\sigma$ [11] is defined by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\Omega)|^2, \tag{1}$$

where $\mathrm{d}\Omega$ is the differential solid angle and $f(\Omega)$ is the scattering amplitude. The Born approximation is known to be very reliable to describe initial and final states of the collision system for high-energy collisions, i. e., $E \gg Ry$ where $E(= \mu v^2/2)$ is the collisions energy, μ the reduced mass of the collision system, v the collision velocity, Ry $(= me^4/2\hbar^2 \cong 13.6\ \text{eV})$ the Rydberg constant, and

m is the electron rest mass. In the first-order Born approximation, the scattering amplitude is given by

$$f(\Omega) = -\frac{\mu}{2\pi\hbar^2}\int \mathrm{d}^3\boldsymbol{r}\, V(\boldsymbol{r}) e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}}, \quad (2)$$

where $V(\boldsymbol{r})$ is the scattering potential, and $\boldsymbol{k}$ and $\boldsymbol{k}'$ are the wave vectors of the incident and scattered wave, respectively.

In the region of partial degenerate and strong coupling, the interaction of charged particles cannot be represented by a pure Coulomb potential but it can be introduced by effective pair potential [14 - 16]. The Kelbg potential [15], obtained by first-order perturbation theory, is known to be a good approximation for two-particle Slater sums in the case of small interaction parameter $\xi[= e^2/(\lambda kT)]$ for large separation of the particle interaction, where $\lambda(= \hbar/\sqrt{2\mu kT})$ is the thermal de Broglie wavelength, k denotes the Boltzmann constant, and T is the plasma temperature. However, the Kelbg potential deviates from the exact value of the Slater sum for small separations. Very recently, the corrected Kelbg potential [12] for the electron-positron interaction was obtained using the Slater sum and its first derivative for small separations and for temperatures $T \geq 10^4$ K. Then, the thermodynamic functions and the pair correlation functions can be described correctly using the corrected Kelbg effective potential obtained from the binary Slater sums. The corrected Kelbg potential for the electron-positron interaction in electron-positron plasmas including all quantum effects (the Heisenberg uncertainty principle and the Pauli exclusion principle) can be shown to be

$$V(r) = -\frac{e^2}{r}\left\{1 - e^{-r^2/\lambda^2} + \sqrt{\pi}\frac{r}{\lambda\gamma}\left[1 - \mathrm{erf}(\gamma r/\lambda)\right]\right\}, \quad (3)$$

where $\lambda = \hbar/\sqrt{mk_\mathrm{B}T}$ is the thermal de Broglie wavelength, $\gamma \equiv (\sqrt{\pi}/\lambda)[e^2/k_\mathrm{B}T \ln S(\xi)]$, $\mathrm{erf}(z) = (2/\sqrt{\pi})\int_0^z \mathrm{d}x\, e^{-x^2}$ is the error function, and the temperature-dependent parameter $S(\xi)$ denotes the binary Slater sum of the electron and positron:

$$S(\xi) = \sqrt{\pi}\xi^3\left[\zeta(3) + \xi^2\zeta(5)\right] + 4\sqrt{\pi}\xi\int_0^\infty \mathrm{d}x \frac{x e^{-x^2}}{1 - e^{-\pi\xi/x}}. \quad (4)$$

Here $\zeta(p)$ denotes the Riemann zeta function [17]. For large separations ($r/\lambda \geq 2$), the corrected Kelbg potential coincides with the Coulomb potential. Thus, the quantum mechanical effects are found to be impotant for small separations, i. e., $r < 2\lambda$ [12]. Here the parameter $S(\xi)$ denotes the binary Slater sum of the electron and positron at zero distance including also symmetric effects coming from the different spin directions, i. e., the Pauli effect. The height of the Kelbg potential at zero-point distance is related to $\ln S(\xi)$ [16] and is not infinite but a finite due to the quantum mechanical effect of diffraction, i. e., the Heisenberg effect. The corrected Kelbg potential has the correct value of the height due to the method of Slater sums. The use of the corrected Kelbg potential (3) allows us to express the scattering amplitude for electron-positron collisions as

$$f(\theta) = a_0\Big[\Big(1 - \frac{\bar{A}}{\sqrt{\pi}\bar{B}}\Big)\frac{1}{\bar{K}^2} + \frac{\bar{A}}{2\bar{B}^2}\Big(\frac{1}{\bar{K}} + \frac{2\bar{B}^2}{\bar{K}^3}\Big)e^{-\bar{K}^2/4\bar{B}^2}\mathrm{erfi}(\bar{K}/2\bar{B}) - \frac{\sqrt{\pi}\bar{\lambda}}{2}\frac{1}{\bar{K}}e^{-\bar{\lambda}^2\bar{K}^2/4}\mathrm{erfi}(\bar{\lambda}\bar{K}/2)\Big]. \quad (5)$$

where θ is the scattering angle measured in the center of mass system, a_0 $(= \hbar/me^2)$ the Bohr radius, $\bar{\lambda}$ $(\equiv \lambda/a_0)$ the scaled thermal de Broglie wavelength, $\bar{A} \equiv \ln S(\xi)/\bar{\lambda}^2$, $\bar{B} \equiv \sqrt{\pi}/\ln S(\xi)$, $\bar{K}$ $[\equiv 2ka_0\sin(\theta/2)] = \sqrt{2\bar{E}}\sin(\theta/2)$ is the momentum transfer, $\bar{E}$ $(\equiv E/Ry)$ the scaled collision energy, and $\mathrm{erfi}(z)$ $[= \mathrm{erf}(iz)/i]$ is the imaginary error function. In collisions of equal mass particles, the differential scattering cross section in the laboratory system can be obtained by the following transformation from the center of mass (CM) system to the laboratory system (lab) [13]:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_\mathrm{lab} = 4\cos\theta_\mathrm{lab}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_\mathrm{CM}, \quad (6)$$

where θ_lab $(= \theta/2)$ is the scattering angle in the laboratory system. The differential electron-positron scattering cross section in units of πa_0^2 in the laboratory system in electron-positron plasmas, taking into account quantum effects, is then found to be

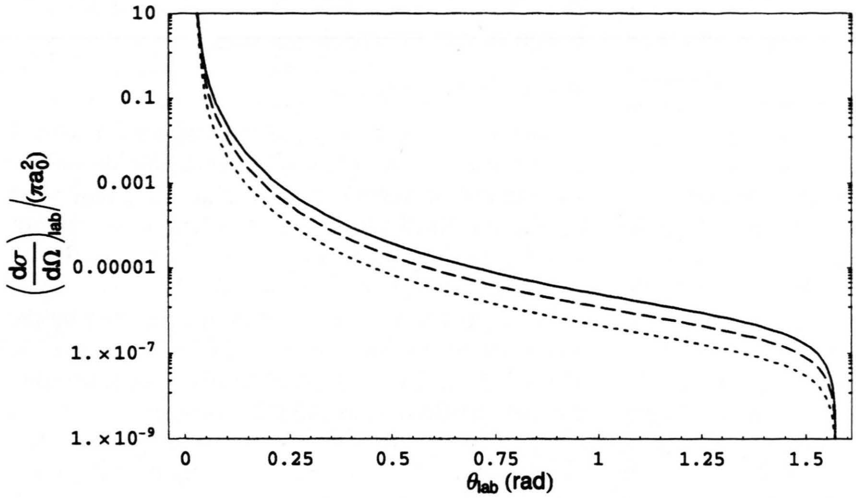

Fig. 1. The differential electron-positron scattering cross sections in units of πa_0^2 as functions of the scattering angle θ_{lab} in the laboratory system in electron-positron plasmas when $\overline{E} = E/Ry = 50$. The solid, dashed and dotted lines represent the electron-positron scattering cross section for T = 10 000 K, 11 000 K, and 12 000 K, respectively.

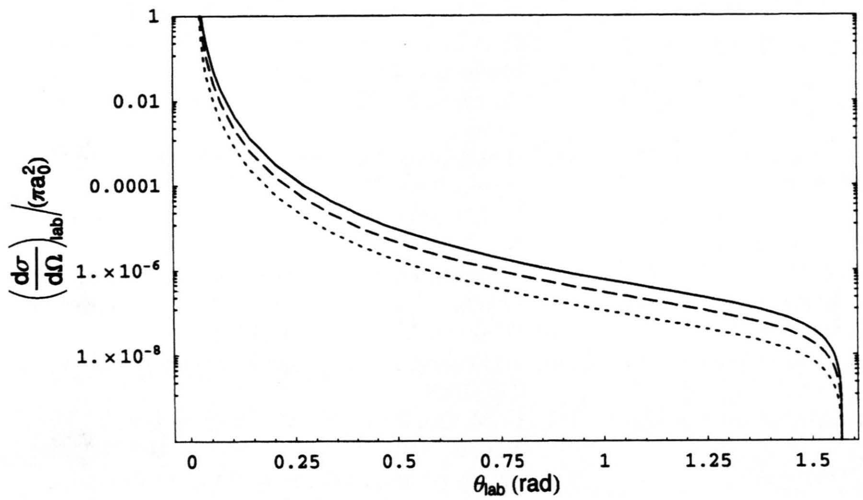

Fig. 2. The differential electron-positron scattering cross sections in units of πa_0^2 as functions of the scattering angle θ_{lab} in the laboratory system in electron-positron plasmas when $\overline{E} = E/Ry = 100$. The solid, dashed and dotted lines represent the electron-positron scattering cross section for T = 10 000 K, 11 000 K, and 12 000 K, respectively.

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\text{lab}}\Big/\pi a_0^2 = \Bigg|\left(1-\frac{\overline{A}}{\sqrt{\pi}\overline{B}}\right)\frac{1}{2\overline{E}\sin^2\theta_{\text{lab}}} + \frac{\overline{A}}{2\overline{B}^2}\left(\frac{1}{\sqrt{2\overline{E}}\sin\theta_{\text{lab}}}+\frac{\overline{B}^2}{\sqrt{2\overline{E}}^{3/2}\sin^{3/2}\theta_{\text{lab}}}\right)e^{-\overline{E}\sin^2\theta_{\text{lab}}/2\overline{B}^2}\operatorname{erfi}(\sqrt{\overline{E}}\sin\theta_{\text{lab}}/\sqrt{2\overline{B}}) - \frac{\sqrt{\pi}\overline{\lambda}}{2}\frac{1}{\sqrt{2\overline{E}}\sin\theta_{\text{lab}}}e^{-\overline{\lambda}^2\overline{E}\sin^2\theta_{\text{lab}}/2}\operatorname{erfi}(\overline{\lambda}\sqrt{\overline{E}}\sin\theta_{\text{lab}}/\sqrt{2})\Bigg|^2. \tag{7}$$

In order to explicitly investigate the quantum effects on the differential electron-positron scattering cross section, specifically, we consider three cases of the thermal de Broglie wavelength: $\overline{\lambda}$ (= λ/a_0) = 5.62, 5.36, and 5.13, i.e., T = 10 000 K, 11 000 K, and 12 000 K, since the corrected Kelbg potential is reliable for $T \geq 10^4$ K. Figures 1 and 2 show the differential electron-positron scattering cross section in units of πa_0^2 as functions of the scattering angle θ_{lab} in the laboratory system in electron-positron plasmas for $\overline{E}$ = 50 and $\overline{E}$ = 100, since the Born approximation is known to be valid for high-energy collisions ($E \gg Ry$). As we see in these figure, the quantum effect significantly increases the electron-positron scattering cross section. The differential scattering cross section increases with decreasing plasma temperature, i. e., increasing de Broglie wavelength. The differential electron-positron scattering cross section decreases with increasing collision energy. Thus it is found that the differential scattering cross section shows the same behaviour of the Rutherford scattering low for the corrected Kelbg potential. It is also found that the quantum effects of the differential scattering cross section are small for the forward ($\theta_{\text{lab}} \approx 0$) and backward scattering angles ($\theta_{\text{lab}} \approx \pi/2$). These results provide useful information of the quantum effects on electron-positron collisions in two-component plasmas.

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